

Smooth deformations and cosmic statefinders

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Abstract

We study the possibility that the universe is subjected to a deformation, besides its expansion described by Friedmann's equations. The concept of smooth deformation of a riemannian manifolds associated with the extrinsic curvature is applied the standard FLRW cosmology. Starting from the resulting modified Friedman's equation we study two possible solutions with six models for each one in low redshift. In other to constrain the models, we calculate deceleration, jerk and Hubble parameters and compare with different data as the latest BAO/CMB + SNIa constraints, SNLS SNIa, x-ray galaxy clusters and the gold sample (SNIa). As a result, we obtain a set of proper models compatible with the current observational data.

Keywords: Embedding, large extra-dimensions, dark energy, cosmology

1. Introduction

The comprehension of the nature of dark matter and dark energy have been considered one of the greatest challenges in contemporary physics. In order to unravel these problems, the understanding of gravitational perturbation has been revealed to be an important issue. Recent observations of the CMBR power spectrum from the Planck mission [3] tell that the gravitational field perturbations amplify the higher acoustic modes due to the gravitational field of baryons. It is conjecture that such perturbations have to do with the influence of dark matter at cosmological scale that would induce a strong gravitational effect leading to a large scale structure formation. Similar effects of gravitational perturbations are also verified at scales of galaxies and clusters, the so-called rotation curve problem.

In the same fashion, gravitational perturbations have influence on the proper explanation of dark energy that requires a more fundamental underlying theory. The Λ CDM model is interpreted as the main model for describing the acceleration expansion of the universe with the equivalence of quantum vacuum energy and cosmological constant. Besides its simplicity and consistency with the current observational data, it ignores the large difference between the very small observed value of the cosmological constant $\Lambda/8\pi G \approx 10^{-47} \text{Gev}^2/c^4$ and the very large averaged value of quantum vacuum energy density $\rho_v \approx 10^{75} \text{Gev}^2/c^4$. Thus, the absence of a feasible solution makes the Λ CDM paradigm an improbable explanation of the accelerated expansion of the universe. This has motivated the emergence of a variety of alternative explanations, including the possible existence of new and previously unheard of essences; the postulation of specific scalar fields; or the possible existence of non observable extra dimensions in space.

In the recent years the extra dimensional proposition seems to be an alternative route to solve the hierarchy of the fundamental interactions: the huge ratio of the Planck to the electroweak energy scale ($M_{Pl}/M_{EW} \sim 10^{16}$). Since Newton's gravitational constant G depends on the dimension of space, then in a higher dimensional space the constant G must change to another value G_* , such that gravitating masses can be correctly evaluated by a (higher dimensional) volume integration of mass densities. However, the existence of extra dimensions must be compatible with the experimentally proven and mathematically consistent four-dimensionality of space-times. This compatibility can be achieved by assuming that the space-times must remain four-dimensional, but they are subspaces of a higher-dimensional space with geometry defined by the Einstein-Hilbert principle. In this case, although the gravitational field propagates in the extra dimensions, the standard gauge interactions and ordinary matter remain confined to the four-dimensional space-time acting as a domain wall [35].

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Several models have been proposed along this direction, mostly belonging to the brane-world paradigm originally proposed in [6], sometimes using additional conditions [31, 32], or other specific embedding assumptions as for example in e.g. [12, 36, 38, 11, 21, 10, 5, 22, 24, 18]. In spite of such efforts we still do not have a complete model independent solution for the present cosmological problems [15]. The only theoretical relation between the extrinsic curvature and the material sources is the Israel-Lanczos boundary condition. However, it is well known that such a condition is of algebraic nature, so that it does not follow the evolution of the brane-world [9]. Some similar models [19, 20] have been developed with no need of particular junction conditions and/or with different junction conditions which lead to several approaches of brane-world models widely studied in literature [25, 1, 37, 40, 16].

In a different approach, Nash's embedding theorem [28] has been revealed to be a powerful tool on the understanding of both embedding process and evolution of a pseudo-riemannian/riemannian geometry (e.g, a braneworld spacetime) by using the concept of smooth deformations of the embedded geometries leading to formation of new ones. In five dimensions, a lesser known principle called Gutpa's theorem [17] is necessary to complement the dynamics of the extrinsic curvature originated from the Gauss-Codazzi equations. Based on the very foundations of geometry, the cosmology of *smooth deformations* has been studied in a previous communication [26]. It was shown that the extrinsic curvature assumed a fundamental role of driving the propagation of gravitation along the extra dimensions of the bulk space and the most evident observable effect is the accelerated expansion of the universe.

In next section, we show the main results of the reference Maia et al., [26]. In the present paper, we explore two possible solutions for the Friedmann's equation called $\gamma^{(+)}$ and $\gamma^{(-)}$ in the low redshift range $0 < z < 2.3$. In the section 3, we present the main purpose of this paper focusing on the study of the Hubble parameter and cosmic statefinders. The main cosmography parameters (deceleration and jerk parameters [42, 43]) to be studied in this work are applied to six models of each γ -solutions. For the resulting deceleration parameters, we compare with the latest BAO/CMB + SNIa constraints [14]. For the resulting jerks, we compare with SNLS SNIa [2], x-ray galaxy clusters [33] and the gold sample (SNIa)[34]. In addition, in order to study the evolution of the Hubble parameter, we compare with the observational data extracted from [8] based on observations of red-enveloped galaxies [39] and BAO peaks [13] supplemented with the observational Hubble parameter data (OHD) with BAO in $\text{Ly}\alpha$ [7, 44]. Finally, in the conclusion section, we present the final considerations.

2. Einstein-Gupta's equations in the FLRW universe

2.1. Theoretical structure

As shown in Maia et al., [26], the Friedmann-Lemaître-Robertson-Walker(FLRW) universe can be regarded as a brane-world in a five-dimensional bulk with constant curvature. The constant curvature bulk is characterized by the Riemann tensor

$$\mathcal{R}_{ABCD} = K_* (\mathcal{G}_{AC}\mathcal{G}_{BD} - \mathcal{G}_{AD}\mathcal{G}_{BC}), \quad A..D = 1 \cdots 5,$$

where \mathcal{G}_{AB} denotes the metric components of the bulk in arbitrary dimensions and $K_* = 6\Lambda$ is either zero(flat bulk) or it can be related to a positive (deSitter) or negative (Anti-deSitter) a bulk cosmological constant.

The cosmological observations [34] suggest that the accelerated expansion scenario is compatible with the deSitter dS_5 space-time. Moreover, the FLRW standard cosmological model is completely embedded in a five-dimensional bulk. Thus, one can obtain the brane-world five-dimensional equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} - Q_{\mu\nu} = -8\pi GT_{\mu\nu}, \quad (1)$$

$$k_{\mu;\rho}^\rho - h_{,\mu} = 0, \quad (2)$$

which are the *gravi-tensor* equation and *gravi-vector* equation, respectively. The Greek indices vary from 1 to 4. The symbol (;) represents the covariant derivative.

The confinement is set simply as $\alpha_* T^* = 8\pi GT_{\mu\nu}$, where $T_{\mu\nu}$ represents the energy momentum tensor of the confined matter, and $T_{\mu 5}^* = T_{55}^* = 0$. Moreover, the quantity $Q_{\mu\nu}$ is given by

$$Q_{\mu\nu} = g^{\rho\sigma} k_{\mu\rho} k_{\nu\sigma} - k_{\mu\nu} H - \frac{1}{2} (K^2 - H^2) g_{\mu\nu}, \quad (3)$$

where $k_{\mu\nu}$ denotes the extrinsic curvature and $H^2 = h \cdot h$, $h = g^{\mu\nu} k_{\mu\nu}$, $K^2 = k^{\mu\nu} k_{\mu\nu}$. It is important to note that the quantity $Q_{\mu\nu}$ is a completely geometrical conserved quantity such that

$$Q_{\mu\nu;\nu} = 0. \quad (4)$$

In principle, the extrinsic curvature can be determined by Codazzi's equation

$$k_{\mu[\nu;\rho]} = 0, \quad (5)$$

where brackets apply to the adjoining indices only. The details in obtaining these equations can be found in [27, 26, 18, 19, 20].

In order to understand the influence of the extrinsic curvature on a FLRW universe in a five-dimensional bulk, we take the line element in (r, θ, ϕ, t) coordinates such that

$$ds^2 = -dt^2 + a^2 \left[dr^2 + f_k^2(r) (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (6)$$

where $f(r)_k = \sin r, r, \sinh r$ that corresponds to spatial curvature $\kappa = 1, 0, -1$, respectively. As the cosmological observations indicate an universe approximately flat, we adopt $k = 0$ and $f_k(r) = r$. The energy-momentum tensor $T_{\mu\nu}$ is the confined source of a perfect fluid in co-moving coordinates given by

$$T_{\mu\nu} = (p + \rho) U_\mu U_\nu + p g_{\mu\nu}, \quad U_\mu = \delta_\mu^4.$$

After solving Codazzi's equations, one can obtain

$$\begin{aligned} k_{ij} &= \frac{b}{a^2} g_{ij}, \quad i, j = 1, 2, 3, \\ k_{44} &= \frac{-1}{a} \frac{d}{dt} \frac{b}{a}, \\ k_{11,\nu} &= 0; \quad k_{11} = b(t), \end{aligned}$$

where $b = b(t) = k_{11}$ is an arbitrary function of time t and $a = a(t)$ is the expansion parameter of the universe. Thus, one can obtain

$$k_{ij} = \frac{b}{a^2} g_{ij}, \quad k_{44} = -\frac{b}{a^2} \left(\frac{B}{H} - 1 \right) g_{44}, \quad i, j = 1..3, \quad (7)$$

$$K^2 = \frac{b^2}{a^4} \left(\frac{B^2}{H^2} - 2 \frac{B}{H} + 4 \right), \quad h = \frac{b}{a^2} \left(\frac{B}{H} + 2 \right), \quad (8)$$

$$Q_{ij} = \frac{b^2}{a^4} \left(2 \frac{B}{H} - 1 \right) g_{ij}, \quad Q_{44} = -\frac{3b^2}{a^4}, \quad i, j = 1..3, \quad (9)$$

$$Q = -(K^2 - h^2) = \frac{6b^2}{a^4} \frac{B}{H}, \quad (10)$$

where the dot denotes the ordinary time derivative, $H = \dot{a}/a$ is the usual Hubble parameter and the extrinsic term $B = \frac{\dot{b}}{b}$ is written in analogy with Hubble parameter.

2.2. The dynamics for extrinsic curvature

It is worth noting that eq.(6) is not perturbed with an additional variable or parameter as commonly seen in most brane-world models. In a different approach, we use the smooth deformations concept based on Nash's embedding theorem where the perturbations can be generally performed on the spacetime itself. It means that the embedded geometry can be warped, bend or stretched. According to Nash theorem [28] the extrinsic curvature generates the perturbations of the gravitational field along the extra dimensions. This lends the physical interpretation that the extrinsic curvature is an independent spin-2 field in the embedded space-time.

On the other hand, in the five-dimensional bulk an additional equation for the extrinsic curvature is required for two reasons: First, the confinement of gauge fields implies that the gravitational vector equations eq.(5) are homogeneous

and the function $b(t)$ does not have a unique solution. Therefore, they are not sufficient to determine completely the extrinsic curvature. Secondly, in the particular case of Minkowski space-time we cannot start Nash's perturbations because the extrinsic curvature of that space-time is zero.

To remedy this situation, a dynamic theory of extrinsic curvature $k_{\mu\nu}$ was proposed [26]. In his paper, Gupta [17] found that any spin-2 field in Minkowski space-time must satisfy an equation that has the same formal structure as Einstein's equations. This result can be obtained by an infinite sequence of infinitesimal perturbations of a linear gravitational equation. Accordingly, he found an Einstein-like system of equations.

As a tentative of extension of Gupta's theorem to a curved spacetime, we found that we could not interpret the extrinsic curvature itself like a metric tensor. The choice of the metric tensor was made as the original first fundamental form $g_{\mu\nu}$. Thus, a new tensor field $f_{\mu\nu}$ was requested to normalize $k_{\mu\nu}$ with process analogous to the "Levi-Civita" connection associated with $f_{\mu\nu}$. As a result, the application of Gupta's theorem led to a "new" geometry for spin-2 field and one can define,

$$f_{\mu\nu} = \frac{2}{K} k_{\mu\nu}, f^{\mu\nu} = \frac{2}{K} k^{\mu\nu}, f_{\mu\nu} f^{\mu\rho} = \delta_\nu^\rho, \quad (11)$$

where $K = K(t)$ is a time-coordinate function.

Hence, an f -connection can be constructed and it can be given by

$$\Upsilon_{\mu\nu\sigma} = \frac{1}{2} (\partial_\mu f_{\sigma\nu} + \partial_\nu f_{\sigma\mu} - \partial_\sigma f_{\mu\nu}), \quad \Upsilon_{\mu\nu}^\lambda = f^{\lambda\sigma} \Upsilon_{\mu\nu\sigma}.$$

It is important to note that the "metric" tensor $f_{\mu\nu}$ and its inverse are used to lower and raise indices on components in the f -geometry. Moreover, a "curvature" tensor for $f_{\mu\nu}$ as

$$\mathcal{F}_{\nu\alpha\lambda\mu} = \partial_\alpha \Upsilon_{\mu\lambda\nu} - \partial_\lambda \Upsilon_{\mu\alpha\nu} + \Upsilon_{\alpha\sigma\mu} \Upsilon_{\lambda\nu}^\sigma - \Upsilon_{\lambda\sigma\mu} \Upsilon_{\alpha\nu}^\sigma,$$

where ∂ denotes the ordinary derivative. Therefore, one can write the "Ricci tensor" and the "scalar tensor", respectively as

$$f^{\alpha\lambda} \mathcal{F}_{\nu\alpha\lambda\mu} = \mathcal{F}_{\mu\lambda\nu}^\lambda = \mathcal{F}_{\mu\nu}, \quad \mathcal{F} = f^{\mu\nu} \mathcal{F}_{\mu\nu}.$$

In addition, using the contracted Bianchi identities, one can find Einstein-Gupta's equations

$$\mathcal{F}_{\mu\nu} - \frac{1}{2} \mathcal{F} f_{\mu\nu} + \Lambda_f f_{\mu\nu} = \alpha_f \zeta_{\mu\nu}, \quad (12)$$

where $\zeta_{\mu\nu}$ is the source of the field $f_{\mu\nu}$, Λ_f and α_f are the cosmological and coupling constants, respectively. It is important to stress that the $f_{\mu\nu}$ metric-type tensor is a geometric field that only exists when the extrinsic geometry is considered. In the 4-dimensional physics such field does not exist for a riemannian observer that only takes into account the tangent vector components. As it happens, to describe a pure gravitational spin-2 system, one can write the Einstein-Gupta's equations as

$$\mathcal{F}_{\mu\nu} = 0. \quad (13)$$

3. Statefinders analysis

When applied to FLRW model in 5-dimensions, Gupta's equation leads to a modified Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8}{3} \pi G \rho + \frac{\Lambda}{3} + \frac{b^2}{a^4}. \quad (14)$$

The contribution of the extrinsic curvature, $b(t)$, is given by

$$b(t) = \alpha_0 (a)^{\beta_0} e^{\mp \frac{1}{2} \gamma(t)}, \quad (15)$$

where all integration constants are combined in α_0 and β_0 . The term $\gamma(t)$ is denoted by

$$\gamma(t) = \sqrt{4\eta_0 a^4 - 3} - \sqrt{3} \arctan\left(\frac{\sqrt{3}}{3} \sqrt{4\eta_0 a^4 - 3}\right). \quad (16)$$

Alternatively, the modified Friedman equation can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{4}{3}\pi G\rho + \frac{\Lambda}{3} + \kappa_0 a^{2\beta_0-4} e^{\pm\gamma(t)} \quad (17)$$

where η_0 and $\kappa_0 = \frac{b_0^2}{a_0^{2\alpha_0}}$ are integration constants. Interestingly, this solution was derived from geometrical approach only, with no assumption of exotic fluids.

3.1. Deceleration parameter

As a starting point, we use the usual form of the deceleration parameter $q(t)$ expressed in terms of the Hubble parameter conveniently written in terms of the redshift z as

$$q(z) = \frac{1}{H(z)} \frac{dH(z)}{dz} (1+z) - 1, \quad (18)$$

where $H(z)$ is the Hubble parameter.

Since we are not considering Λ as the main cause of the accelerated expansion, in the following, we neglect the cosmological constant Ω_Λ and Ω_κ curvature density parameters, and write

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{ext} (1+z)^{4-2\beta_0} e^{\pm\gamma(z)}}, \quad (19)$$

where H_0 is the current Hubble constant. It is important to note that we have two cases to study according to eq.(19) which we call for simplicity γ^+ and γ^- solutions. The matter density parameter is denoted by Ω_m . The term Ω_{ext} stands for the density parameter associated with the extrinsic curvature. Thus, we express (16) in terms of the redshift

$$\gamma(t) = A(z) - \sqrt{3} \arctan\left(\frac{\sqrt{3}}{3} A(z)\right). \quad (20)$$

where $A(z) = \sqrt{\frac{4\eta_0}{(1+z)^4} - 3}$. Notice that $\gamma(z)$ must be real so that we must have the condition

$$\eta_0 \geq 0.75(1+z)^4. \quad (21)$$

As a natural start point, based on the fact that the equal sign in the eq.(21) holds for $\pm\gamma(z) = 0$, this corresponds to the phenomenological solution found in [27] that mimics the X-CDM model with a correspondence

$$4 - 2\beta_0 = 3(1+w), \quad (22)$$

where the parameter w holds for the exotic fluid parameter for an X-fluid [41]. For instance, motivated by the latest Planck observations, for Λ CDM (with $w = -1$) and phantom models ($w < -1$) we have $\beta_0 = 2$ and $\beta_0 = 3$, respectively. It is worth noting that this correspondence is not a necessary condition (but an easier way) to constrain the parameters. This first prior constraint can avoid a larger error propagation from observational data (in the case of using χ -square fitting as shown in the study of high-order statefinders [23]). We are interested in solutions $\pm\gamma \neq 0$ in the low-redshift $[0, 2.3]$ where we have more reliable data.

In order to test the model, we need to constraint the two parameters (η_0, β_0) . In the course of this study, we notice that β_0 affects the value of current deceleration parameter q_0 and η_0 rules mainly on the width of the transition phase z_t . For the β_0 parameter, using eq.(22) we have found the constraint

$$2 \leq \beta_0 \leq 3. \quad (23)$$

Tabela 1: Six models studied for selected values of (η_0, β_0) and the predicted current deceleration parameter q_0 for both $\gamma^{(+)}$, $\gamma^{(-)}$ solutions.

γ -solutions	$\gamma^{(+)}$		$\gamma^{(-)}$	
Parameters	η_0	β_0	q_0	q_0
Model				
I	0.001	2	-0.549	-0.556
II	0.1	2	-0.460	-0.640
III	0.25	2	-0.341	-0.759
IV	0.5	2	-0.362	-0.738
V	0.1	2.5	-0.810	-0.996
VI	0.1	3	-1.160	-1.340

To constrain the values of η_0 , we impose that it must be restricted from its current value (i.e, for the redshift $z = 0$) to the asymptotic value of the deceleration parameter (i.e, for the redshift $z = -1$). In these terms we have

$$0 \leq \eta_0 \leq 0.75 . \quad (24)$$

This result does not contradict eq.(21).

Concerning the cosmological density parameters and using the normalization $H|_{z=0} = H_0$, we fix the matter contribution to be $\Omega_m = 0.3$. For the present epoch,

$$\Omega_{total}|_{z=0} = \Omega_{ext} \exp(\gamma(0)) + \Omega_m = 1 .$$

The density parameter associated with the extrinsic curvature can be given by

$$\Omega_{ext} = \frac{1 - \Omega_m}{\exp(\gamma(0))} .$$

Moreover, using the definition in eq.(18), we obtain the deceleration parameter

$$q(z) = \frac{3}{2} \left[\frac{\Omega_m(1+z)^3 + \gamma^* \Omega_{ext}(1+z)^{4-2\beta_0} e^{\mp\gamma(z)}}{\Omega_m(1+z)^3 + \Omega_{ext}(1+z)^{4-2\beta_0} e^{\mp\gamma(z)}} \right] - 1 . \quad (25)$$

where $\gamma^* = \frac{1}{3} \left[4 - 2\beta_0 \pm 2 \sqrt{\frac{4\eta_0}{(1+z)^4} - 3} \right]$. In addition, using eq.(25) with the constrained values of β_0 in eq.(23) for both solutions $\{\gamma^{(+)}, \gamma^{(-)}\}$, we find that the values of $\eta_0 \geq 0.5$ lead to an incompatible pattern as expected for deceleration parameter. Thus, the constraints on η_0 can be tighter and written as

$$0 \leq \eta_0 \leq 0.5 . \quad (26)$$

In table (1), based on eq.(23) and eq.(26), we study six models for the cases $\{\gamma^{(+)}, \gamma^{(-)}\}$. Hereafter, in order to facilitate the visualization, all models of both γ -solutions are represented by the same type of specific curve. The Λ CDM model is represented by a solid line. Accordingly, Models I, II, III, VI, V and VI are presented by lines with short-dot, dash, dash-dot-dot with an up-triangle, dash-dot with a circle, dot and dash with a star, respectively. It is important to point out that what was intended in defining each model, with a certain value of (η_0, β_0) , was varying its possible values with its respective ranges as shown in eqs.(26) and (23). For instance, the models V and VI are defined with $\eta_0 = 0.1$ and $\beta_0 = 2.5$ and $\beta_0 = 3$, respectively. Varying η_0 from 0.1 to 0.5, we do not verified a considerable difference in the results. In this respect, with the proposed models in course, one can expect to obtain a general view of the applicability of this theoretical scheme.

The resulting deceleration parameter for $\gamma^{(+)}$ is shown in the left figure in fig.(1).

The model I is in concordance with Λ CDM (solid line). It is worth noting that the models from II to V are also in concordance with BAO/CMB + SNIa [14]. The model VI presents a phantom-like pattern. For these models, the transition redshift z_t lies at $0.57 \leq z_t \leq 0.74$.

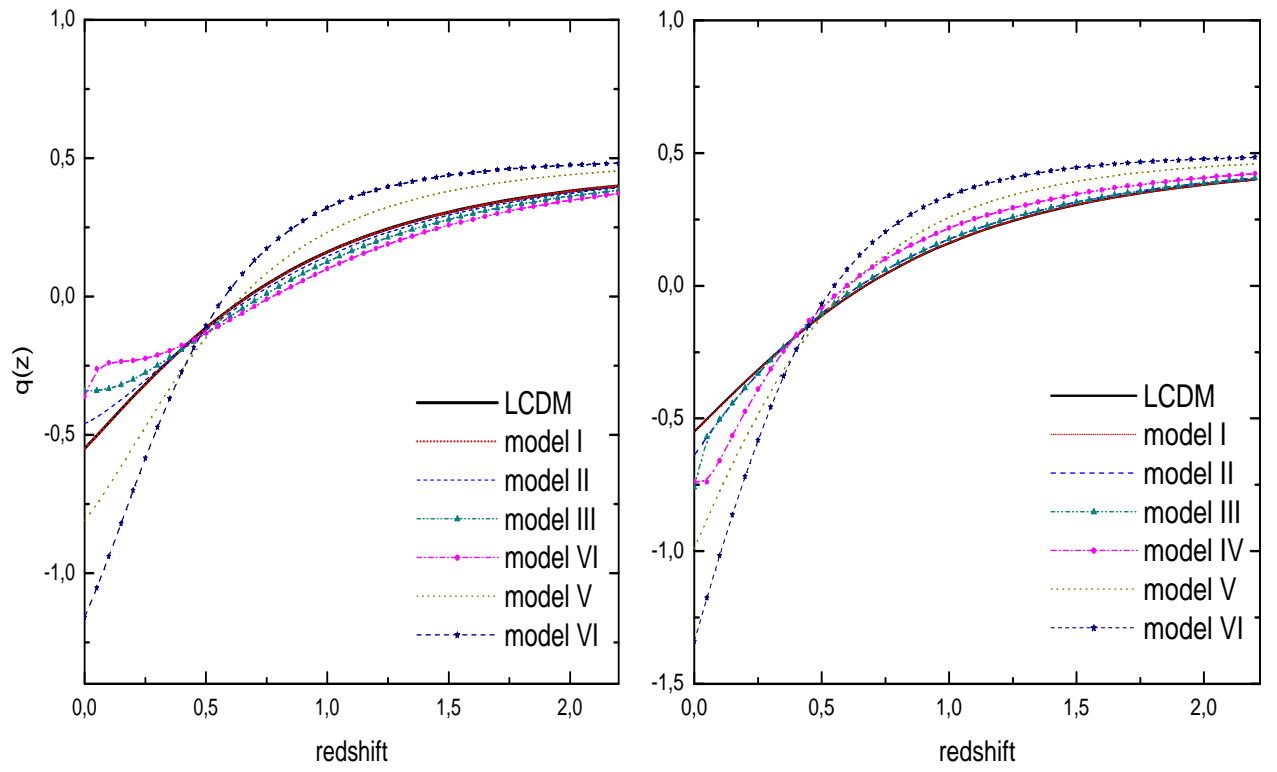


Figura 1: Deceleration parameter as a function of redshift for solutions of $\gamma^{(+)}$ (left figure) and $\gamma^{(-)}$ (right figure). From each model, the resulting current deceleration parameter q_0 is also indicated.

On the other hand, the γ^- solution predicts models with more intense acceleration as compared with the previous case as it can be shown in the right figure in fig.(1). Again, the model I is in completely concordance (overlapped) with Λ CDM and no significant differences can be observed. It is worth noting that the models I to V are in concordance with BAO/CMB + SNIa [14]. The model VI presents a phantom-like pattern. For these models, the transition redshift z_t lies at $0.55 \leq z_t \leq 0.68$.

According with BAO/CMB + SNIa constraints, with MLCS2K2 light-curve fitter it gives $z_t = 0.56^{+0.13}_{-0.10}$ and $q_0 = -0.31^{+0.11}_{-0.11}$ with 68% C.L favors DGP-like models. With SALT2 fitter, $z_t = 0.64^{+0.13}_{-0.07}$ and $q_0 = -0.53^{+0.17}_{-0.13}$ with 68% C.L that favors Λ CDM models. For both solutions $\{\gamma^{(+)}, \gamma^{(-)}\}$, the transition z_t mimics DGP-like models as well as Λ CDM models. As table (1) indicates for $\gamma^{(+)}$ solution, models I and II mimic Λ CDM-like models. The model III and IV mimic DGP-like models. Finally, the model V and VI indicate a quintessence-like and phantom-like behavior, respectively.

In addition, for $\gamma^{(-)}$ solution, models I and II also mimic Λ CDM-like models. Differently to the previous $\gamma^{(+)}$ case, for the $\gamma^{(-)}$ solution for the models III, IV and V, they mimic a quintessence-like models. Finally, the model VI also indicates a phantom-like behavior just like in the case of $\gamma^{(+)}$.

It is worth noting that in both cases $\{\gamma^{(+)}, \gamma^{(-)}\}$, the models I and II mimic Λ CDM-like models according with BAO/CMB + SNIa constraints. This was expected with $\beta_0 = 2$ and a small η_0 close to zero. A similar expectation have occurred to obtain quintessence-like and/or phantom-like behaviors when we set β_0 close or equal to 3 which we have confirmed with the models V and VI (for both γ -solutions) with a more negative current value for the deceleration parameter. On the other hand, in order to select models even more constrained, we need to analyze the jerk parameter as shown in the next section.

3.2. Jerk parameter

The jerk parameter j is defined as the third time-order derivative of the scale factor a and is given by

$$j = \frac{\ddot{a}}{aH^3} . \quad (27)$$

If one takes a Taylor expansion of the scale factor around its current value a_0 ,

$$\begin{aligned} \frac{a(t)}{a_0} &= 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 + \\ &\quad \frac{1}{6}j_0H_0^3(t - t_0)^3 + \mathcal{O}[(t - t_0)^4] + \dots, \end{aligned}$$

and eq.(27) can be rewritten as [30]

$$j = q + 2q^2 - \frac{1}{H} \frac{dq}{dz} , \quad (28)$$

where we denote $j = j(z)$ and $q = q(z)$.

Since the hubble parameter $H(z)$ depends on the differential age as a function of the redshift such as

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} ,$$

one can write the jerk parameter in terms of redshift as

$$j = q + 2q^2 + (1+z) \frac{dq}{dz} . \quad (29)$$

Another feature of the jerk parameter is that it can be used to select gravitational models. Using eq.(29), we obtain fig.(2) for the solutions $\gamma^{(+)}$ and $\gamma^{(-)}$, respectively. The pair (η_0, β_0) for each γ solutions are the same as used in table (1).

We compare now with the SNLS SNIa dataset that gives $j = 1.32^{+1.37}_{-1.21}$ [2] x-ray galaxy clusters $j = 0.51^{+2.55}_{-2.00}$ [33] and the gold sample (SNIa) [34] that gives $j = 2.75^{+1.22}_{-1.10}$. For $\gamma^{(+)}$ solution in the left figure in fig.(1), the models I, II and V are compatible with all the previous datasets. With a possible negative jerk, model 3 are compatible with the

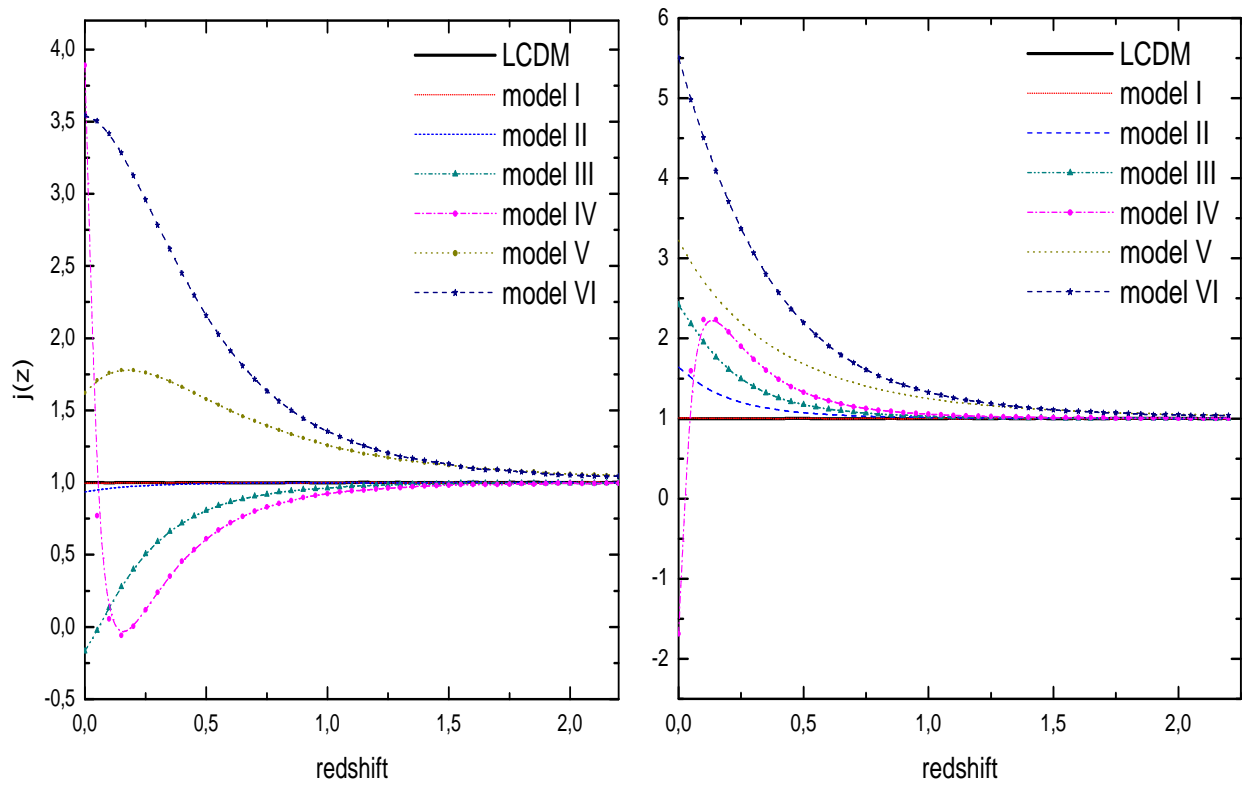


Figura 2: Jerk parameter as a function of redshift for solutions of $\gamma^{(+)}$ (left figure) and $\gamma^{(-)}$ (right figure).

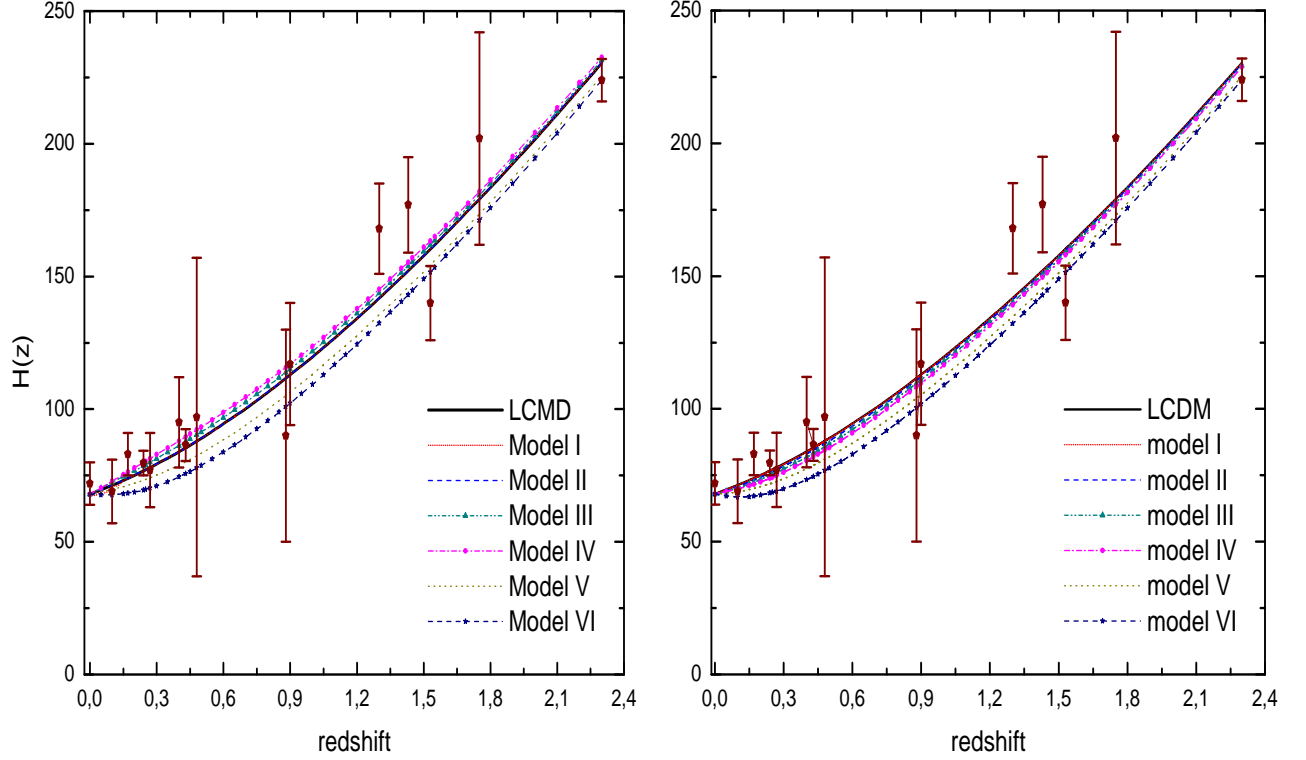


Figure 3: Deceleration parameter as a function of redshift for solutions of $\gamma^{(+)}$ (left figure) and $\gamma^{(-)}$ (right figure). Error bar points were extracted from [8] supplemented with additional from [7, 44] at $H(z = 2.3)$.

x-ray galaxy cluster only. The model VI are compatible with the gold sample (SNIa) only. For $\gamma^{(-)}$ solution in the right figure in fig.(1), the models I, II and III are compatible with all the previous datasets. The model V is compatible with both x-ray galaxy clusters and the gold sample (SNIa). The model VI is incompatible with observations and the phantom-like behavior is not allowed, at least in $\gamma^{(-)}$ solution.

The model IV fails in both γ -solutions and can be neglected. A common feature of all models is that for redshift $z < 2$ all the curves tend to the Λ CDM-like pattern.

3.3. Hubble parameter

We also study the evolution of the hubble parameter $H(z)$ in the redshift range $0 < z < 1.8$ with original data observational points extracted from [8] based on observations of red-enveloped galaxies [39] and BAO peaks [13]. It was also supplemented with the observational Hubble parameter data (OHD) and BAO in Ly α [7, 44] implying that $H(z = 2.3) = (224 \pm 8) \text{ km.s}^{-1} \cdot \text{Mpc}^{-1}$. In this respect, we obtain the following curves in the fig.(3).

For both cases $\gamma^{(+)}$ and $\gamma^{(-)}$, we obtain a similar pattern very close to Λ CDM-like behavior for the first four models of both γ -solutions. In $\gamma^{(-)}$ solution, the model III and IV indicate a lower values for Hubble parameter as compared with the same models in $\gamma^{(+)}$ solution. This characteristic can be interesting to further studies on primordial nucleosynthesis. The models V and VI present the lowest values for Hubble parameter as compared with all models studied but still with agreement with observations [7]. The overall conclusion is due to the results of the curves of jerk parameters, the model IV must be neglected in both γ -solutions which means that the η_0 parameter can be constrained to the range $0 \leq \eta_0 < 0.5$. In the same sense, the model VI in $\gamma^{(-)}$ solution can be neglected. Moreover, the other models present a good concordance with observations and can be explored as a package for cosmological purposes and more constrained with improved techniques of near-future observations.

4. Final remarks

We have applied the concept of smooth deformations of riemannian manifolds to space-times and in particular to the FLRW universe, showing that it is an efficient mechanism to explain the accelerated expansion of the universe. In such geometric process, some of the topological informations of space-times does not appear in riemannian geometry. For instance, the conserved quantity $Q_{\mu\nu}$ may be interpreted as a component of some mechanical energy responsible for the observed acceleration of the universe. However, from the point of view of geometry, it may be also interpreted as a necessary observational quantity, reintroducing some topological qualities to a gravitational theory. This latter interpretation gives full support to the Gauss and Riemann views that geometry is determined essentially by the observations, regardless how small and near or how large and distant as they may be.

The acceleration of the universe was described as a consequence of the extrinsic curvature of the space-time embedded in a bulk space, defined by the Einstein-Hilbert action. It seems natural that this result provides the required geometrical structure to describe a dynamically changing universe. The four-dimensionality of the embedded space-times was determined by the dualities of the gauge fields, which corresponds to the equivalent concept of confinement gauge fields and ordinary matter in the brane-world program. However, this confinement implies that the extrinsic curvature cannot be completely determined, simply because Codazzi's equations becomes homogeneous. Since the extrinsic curvature assumes a fundamental role in Nash's theorem, an additional equation was required. We have noted that the extrinsic curvature is an independent rank-2 symmetric tensor which corresponds to a spin-2 field defined on the embedded space-time. However, as it was demonstrated by Gupta, any spin-2 field must satisfy an Einstein-like equation.

After the due adaption to an embedded space-time, in a previous work [26] we have constructed Einstein-Gupta's equations for the extrinsic curvature of the FLWR geometry. Here we have extended the idea to the study of the behavior of its solution at low redshift. This was compared with a phenomenological model (Λ CDM), obtaining very similar patterns but using a geometrical approach. However, two models in $\gamma^{(\pm)}$ -solutions were neglected constrained in the analysis of the jerk parameter revealing a incompatibility with the observational data.

It is important to stress that all different datasets presented were not used to obtain the results presented in this paper. On the contrary, they were used to constrain the models studied only. As future perspectives, the models presented must be studied with the analysis on high redshift and primordial nucleosynthesis. Also, the snap and lerk parameters must be contemplated and should be investigated further.

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